



Stabilization of a CSTR with two arbitrarily switching modes using modal state feedback linearization

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ABSTRACT

The problem of controller synthesis with the objective of stabilizing a continuous stirred tank reactor (CSTR) with arbitrary switching between two modes, is considered. First, based on the new concept of modal state feedback linearization, two nonlinear state feedback laws and a nonlinear state transformation are synthesized. The advantage of this step is to transform the switched nonlinear model of the CSTR to an equivalent switched linear system without any approximation. In the second step, a stabilizing controller is designed for the switched linear system using the common Lyapunov function theory. Although it is proven that the process is globally stabilized with the designed controller, the performance of the controller is also shown in simulation. This paper illustrates the possibility of simplifying the procedure of designing controller for switched nonlinear processes, using the modal state feedback method.

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1. Introduction

Many chemical processes include discontinuous actuators, physical constraints or manufacturing distinct phases such as, filling/emptying a reactor or heating/cooling a product (see for example [1–5]). Drastic, instantaneous changes in a continuous behavior of a process, caused by such factors, can be modelled more conveniently as discrete events. In many cases, discrete and continuous dynamics of the process interact to such a significant extent that they cannot be decoupled effectively. This characteristic would complicate the modelling, analysis and design of such processes.

Hybrid system framework is general enough to model processes where the behavior of interest is determined by interacting continuous and discrete dynamics and state jumps [6]. Switched systems are a special class of hybrid systems. A switched system consists of several subsystems (modes) and a switching signal that specifies the active subsystem at each time instant. Switched systems can be categorized as switched linear systems (SLSs) and switched nonlinear systems (SNSs). It should be noted that even an SLS which consists of linear subsystems is a nonlinear system. Research on switched systems has been an active field during recent years. For a survey on switched systems we refer to [1,7–11].

The control of switched systems is a challenging issue. Motivated by stability analysis and stabilization of switched systems, many interesting problems have been investigated in recent years [8,10]. One of these problems is the stabilization of a switched sys-

tem under arbitrary switching signal. It is well established [8,10] that if a common Lyapunov function (CLF) exists for the constituent systems of a switched system, then the system is asymptotically stable under arbitrary switching signal.

Much of the recent research on the stability of SLSs under arbitrary switching signal is concerned with obtaining verifiable conditions that guarantee the existence of a CLF for constituent linear systems. Some mature results in this area have been presented (see [10] and references there in). For SNSs, however, there are only some limited results on the similar issue [12–14], and this problem, in general, is yet far from understood.

Continuous stirred tank reactors (CSTRs) are known to be one of the systems that exhibit complex behavior. Previously, linear control approaches which are derived on the basis of linearized models of the process have been applied to CSTRs [15–17]. However, CSTRs are difficult to control effectively using linear techniques due to their inherent nonlinear behavior. The other source of complexity is that it is often desirable to operate CSTRs in an open-loop unstable region due to the suitable reaction behavior there. Therefore, nonlinear design tools such as feedback linearization have been used to provide global stabilization [18,19]. Also, various auxiliary solutions have been proposed to overcome inherent drawbacks of the feedback linearization approach. In particular, state observers have been designed to estimate non-measurable states [20–23]. Also robust techniques have been utilized to reduce the effect of parameter uncertainties [24–26]. Moreover, input constraints and multivariable behavior of CSTRs, encourage the utilization of other advanced controllers (see, e.g., [27–31]).

In this paper, we consider a CSTR with a hybrid behavior as a case study. It can be modelled as an SNS. Although many

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Nomenclature

a_0, a_1, a_2	constant coefficients
A	system matrix
b	input vector
B	input matrix
C_A	reactant A concentration (mol L^{-1})
C_p	heat capacity of the fluid ($\text{J g}^{-1}\text{K}^{-1}$)
E	activation energy (J mol^{-1})
k_0	reaction rate constant (min^{-1})
q	feed flow rate (L min^{-1})
R	gas constant ($\text{J mol}^{-1}\text{K}^{-1}$)
t	time (min)
T	reactor temperature (K)
T_c	coolant temperature (K)
u	switched nonlinear system input
UA	heat transfer constant ($\text{J min}^{-1}\text{K}^{-1}$)
v	switched linear system input
V	volume of the reactor (L)
x_1, x_2	switched nonlinear system states
z_1, z_2	switched linear system states

Greek letters

ΔH	enthalpy of the reaction (J mol^{-1})
ρ	density of the fluid (g L^{-1})
Σ_A	linear system with A matrix

Superscripts/subscripts

*	nominal operating conditions
0	initial conditions
e	equilibrium
f	feed stream index
i	mode index

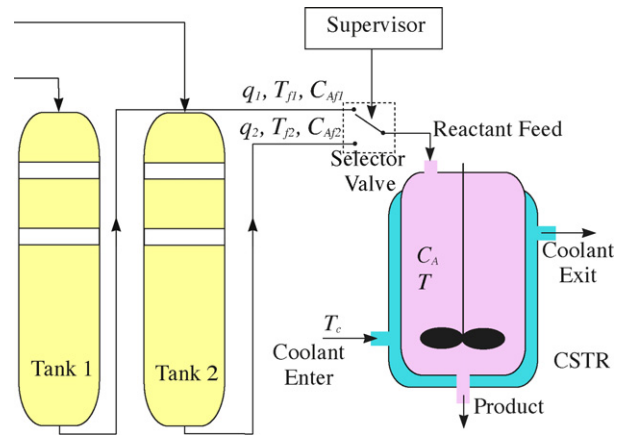


Fig. 1. Schematic diagram of the process.

Table 1

Nominal parameters of the process [37].

V (L)	100
ρ (g L^{-1})	1000
C_p ($\text{J g}^{-1}\text{K}^{-1}$)	0.239
ΔH (J mol^{-1})	-5×10^4
E/R (K)	8750
k_0 (min^{-1})	7.2×10^{10}
UA ($\text{J min}^{-1}\text{K}^{-1}$)	5×10^4
$a_0 = k_0$	7.2×10^{10}
$a_1 = \frac{\Delta H}{\rho C_p} k_0$	-1.506×10^{13}
$a_2 = \frac{UA}{V \rho C_p}$	2.092

form $A \rightarrow B$ occurs. The reactor is cooled by a coolant stream with a constant flow rate and a variable temperature T_c .

advanced control techniques have been developed for CSTRs, none of them can deal effectively with a CSTR which has a switched model. The stabilization of SNSs is still a challenging problem and only some limited results have been reported in the literature, such as [32–36]. The main contribution of this work is to demonstrate the development of the modal feedback linearization technique, introduced in [36], for a CSTR with two modes. Modal feedback linearization will be utilized to transform the switched nonlinear model of the process to an equivalent switched linear model without resorting to any approximation. Then a control law will be designed based on the resulted switched linear model to provide global stabilization under arbitrary switching signal.

The outline of the paper is as follows: The process model is presented in Section 2. In Section 3 the theoretical problem is formulated and solved. Simulation results are illustrated and discussed in Section 4. Finally, a conclusion is drawn in Section 5.

2. Process description

The process is shown schematically in Fig. 1. It consists of a constant volume CSTR fed by a single inlet stream through a selector valve which is connected to two different source streams. Suppose that the position of the selector valve at each time is determined by a supervisory mechanism based on an objective. In other words, at each time the reactor is fed by one of the source streams according to the decision made by the supervisor. Since the source streams have different parameters, the parameters of the feed of the reactor change instantaneously and the reactor will have two operating modes. In the reactor an exothermic, irreversible reaction of the

2.1. Mathematical model

Assuming constant liquid volume, negligible heat losses, perfectly mixing and a first-order reaction in reactant A, the CSTR at each operating mode is described by the following differential equations:

$$\begin{aligned} \dot{C}_A &= \frac{q_i}{V} (C_{Af_i} - C_A) - a_0 \exp\left(-\frac{E}{RT}\right) C_A, \\ \dot{T} &= \frac{q_i}{V} (T_{f_i} - T) - a_1 \exp\left(-\frac{E}{RT}\right) C_A + a_2 (T_c - T). \end{aligned} \quad (1)$$

These equations and the nominal values of the parameters which can be found in Table 1 are described in [37]. As stated above, the reactor has two modes with respect to the feeding stream. The parameters which are considered for the feed streams are indicated in Table 2. The nominal operating conditions corresponding to an unstable equilibrium point are $T_c^* = 300$ K, $C_A^* = 0.5$ mol/L and $T^* = 350$ K for both modes.

In this paper, the supervisory mechanism is not considered in the model. Instead, it is assumed that the position of the selector valve is determined by an arbitrary signal and this signal which determines the mode of the reactor is known at each time.

Table 2

Parameters of the feed stream.

mode	q (L min^{-1})	C_{Af} (mol L^{-1})	T_f (K)
1	50	1.5	350
2	200	0.75	350

Defining the states $x_1 = C_A - C_A^*$, $x_2 = T - T^*$ and the control input $u = T_c - T_c^*$, system (1) can be written in the form of SNS

$$\dot{x}_1 = f_1^i(x_1, x_2) + g_1^i(x_1, x_2) u, \quad (2)$$

$$\dot{x}_2 = f_2^i(x_1, x_2) + g_2^i(x_1, x_2) u,$$

with $i \in \{1, 2\}$, and

$$f_1^i = \frac{q_i}{V} (C_{Afi} - C_A^* - x_1) - a_0 (x_1 + C_A^*) \exp\left(-\frac{E/R}{x_2 + T^*}\right), \quad (2a)$$

$$f_2^i = \frac{q_i}{V} (T_{fi} - T^* - x_2) - a_1 \exp\left(-\frac{E/R}{x_2 + T^*}\right) (x_1 + C_A^*) + a_2 (T_c^* - x_2 - T^*), \quad (2b)$$

$$g_1^i = 0, \quad (2c)$$

and

$$g_2^i = a_2. \quad (2d)$$

In the SNS (2) the discrete state value, i , represents the arbitrary signal which determines the position of the selector valve or equivalently the reactor mode.

2.2. Control objective

Suppose that the values of system states, T and C_A , and the discrete state value which is the selector valve position, are available at each time. The control objective is to regulate C_A and T to their nominal values by manipulating T_c under an arbitrary switching of the selector valve position.

It is well established that the stability of individual modes do not guarantee the stability of a switched system under arbitrary switching [9]. Therefore, the design of a stabilizing controller for each mode is not sufficient to fulfill our control objectives. Auxiliary techniques should be developed to guarantee the stability under arbitrary switching signal.

3. Theory and design

In this section, the design of a stabilizing controller is presented for the switched nonlinear model of the CSTR process described in Section 2. The design consists of two main stages. In the first stage, based on the new concept of modal state feedback linearization [36], the SNS is transformed into an equivalent SLS. Then, in the second stage, a stabilizing controller is designed for the equivalent SLS. The final control law is calculated by combining the results of these two stages.

3.1. Modal state feedback linearization

There are mature results for some of the stabilizing problems considering SLSs, while most of the similar problems considering SNSs remain unsolved [10,38,32]. For SNSs, however, modal state feedback linearization could be a useful concept. This approach is applicable to the modal state feedback linearizable systems.

Definition 1. Consider an input affine SNS

$$\dot{x} = f^i(x) + g^i(x) u, \quad (3)$$

where $i \in I = \{1, \dots, M\}$ is the subsystems' index and $g^i(x_e) \neq 0$, $\forall i \in I$. We assume that $(x_e, u_e) = (0, 0)$ is the common equilibrium point for all the subsystems. The SNS (3) is said to be locally modal state feedback linearizable, if there exist a locally diffeomorphism state transformation $z = T(x)$, where $T(0) = 0$, and smooth function

pairs $(f_x^i(x), g_x^i(x))$ with $f_x^i(0) = 0$, $g_x^i(0) \neq 0$, $\forall i \in I$, defining for each subsystem a state feedback

$$u = \frac{-f_x^i + v}{g_x^i}, \quad (4)$$

such that the closed-loop system in z coordinates becomes

$$\dot{z} = A^i z + B^i v, \quad (5)$$

where (5) is a controllable SLS in the sense of [39].

The motivating ideas behind both the modal state feedback linearization and the conventional state feedback linearization [40] are almost similar. However, due to some basic differences, similar techniques cannot be applied to these two problems [36]. Finding the modal state feedback linearizability conditions, in a general case, is an open problem. This problem is formulated and solved for some classes of second-order SNSs in [36]. One of these classes is the SNS (3) which can be represented by the model

$$\begin{cases} \dot{x}_1 = p_1^i f_{11}(x_1) + p_2^i f_{12}(x_1, x_2) \\ \dot{x}_2 = f_2^i(x_1, x_2) + g_2^i(x_1, x_2) u \end{cases} \quad (6)$$

where $p_1^i, p_2^i \in \mathbb{R}$ are parameters of the system and f_{11} and f_{12} are linearly independent.

Consider the switched nonlinear model of the process in Section 2. This model can be represented as (6) with $p_1^i = -q_i/V$, $p_2^i = 1$, $f_{11} = x_1$, and

$$f_{12} = \frac{q_i}{V} (C_{Afi} - C_A^*) - a_0 \exp\left(-\frac{E/R}{x_2 + T^*}\right) (x_1 + C_A^*).$$

Then, following [36] Theorem 3 and Corollary 2, it can be easily shown that the switched nonlinear model of the process is locally modal state feedback linearizable with

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ \frac{q_i}{V} (C_{Afi} - C_A^*) - a_0 \exp\left(-\frac{E/R}{x_2 + T^*}\right) (x_1 + C_A^*) \end{bmatrix} \quad (7)$$

as a diffeomorphism transformation and the state feedback (4) with

$$f_x^i = -a_0 \exp\left(-\frac{E/R}{x_2 + T^*}\right) \times \left[\frac{q_i}{V} (C_{Afi} - C_A^*) - a_0 (x_1 + C_A^*) \exp\left(-\frac{E/R}{x_2 + T^*}\right) \right] - a_0 \exp\left(-\frac{E/R}{x_2 + T^*}\right) \left(\frac{E/R}{(x_2 + T^*)^2} \right) (x_1 + C_A^*) \times \left[\frac{q_i}{V} (T_{fi} - T^* - x_2) - a_1 (x_1 + C_A^*) \exp\left(-\frac{E/R}{x_2 + T^*}\right) - a_2 (T_c^* - x_2 - T^*) \right] \quad (8)$$

and

$$g_x^i = g_x^2 = -a_0 a_2 \left(\frac{E/R}{(x_2 + T^*)^2} \right) \exp\left(-\frac{E/R}{x_2 + T^*}\right). \quad (9)$$

The diagram of the proposed control system is presented in Fig. 2. Applying the state feedbacks and the state transformation, result in

$$\Sigma_1 : \begin{cases} \dot{z}_1 = 0.5z_1 + z_2 \\ \dot{z}_2 = u \end{cases}; \quad \Sigma_2 : \begin{cases} \dot{z}_1 = 2z_1 + z_2 \\ \dot{z}_2 = u \end{cases}, \quad (10)$$

where (10) is a controllable SLS.

3.2. Stabilizing control law

As mentioned in Section 2, the control objective is to regulate states of the process to the desired equilibrium point under arbitrary switching of the selector valve. It can be easily verified that, if the resulted SLS in Section 3.1 is asymptotically stabilized under

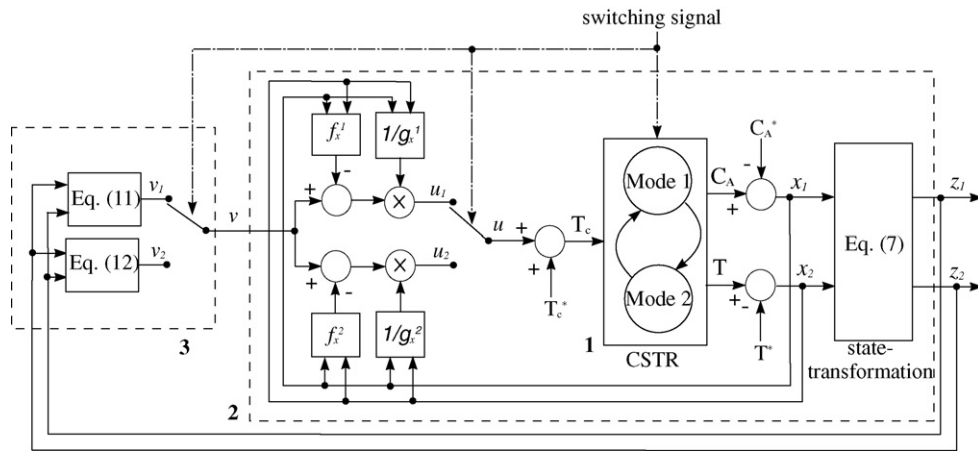


Fig. 2. Diagram for the control system. (1) CSTR process with two modes, (2) the SLS resulted from the modal state feedback linearization, (3) the stabilizing controller for the SLS.

arbitrary switching signal, the process will be stabilized and consequently the control objective will be met.

Many approaches to the problem of stability of SLSs under arbitrary switching signal rely on the construction of common quadratic [10,41,42] or non-quadratic [43–45] Lyapunov functions for the constituent subsystems. In spite of the fact that the quadratic Lyapunov function as a special structure for Lyapunov function may limit the results, the simplicity that this form provides encourages many researchers to focus on it. The following theorem is useful for our case.

Theorem 2. [41]. Let A_1 and A_2 be Hurwitz matrices in $\mathbb{R}^{2 \times 2}$. Then two LTI systems Σ_{A_1} and Σ_{A_2} have a common quadratic Lyapunov function (CQLF) if and only if the matrices $A_1 A_2$ and $A_1 A_2^{-1}$ do not have real negative eigenvalues.

Theorem 2 provides a simply verifiable, necessary and sufficient condition for the existence of a CQLF for a pair of second order LTI system. In our case, we can design linear controllers to satisfy the

theorem's condition. Considering state feedback controllers

$$v_1 = -k_{11}z_1 - k_{12}z_2, \tag{11}$$

$$v_2 = -k_{21}z_1 - k_{22}z_2, \tag{12}$$

the closed-loop subsystems become

$$A_{1c} = \begin{bmatrix} 0.5 & 1 \\ -k_{11} & -k_{12} \end{bmatrix},$$

$$A_{2c} = \begin{bmatrix} 2 & 1 \\ -k_{21} & -k_{22} \end{bmatrix}.$$

A_{1c} and A_{2c} have a CQLF if and only if these two matrices are Hurwitz and all of the eigenvalues of $A_{1c}A_{2c}$ and $A_{1c}A_{2c}^{-1}$ are complex or real positive. These conditions can be formulated as a set of inequalities which constrain the state feedback gains. Some of these inequalities are nonlinear and their explicit solving is impossible. However, it is possible to find sets of state feedback gains which satisfy all the inequalities. For example, $k_{11} = 3$, $k_{12} = 3$, $k_{21} = 8$ and $k_{22} = 3$ is

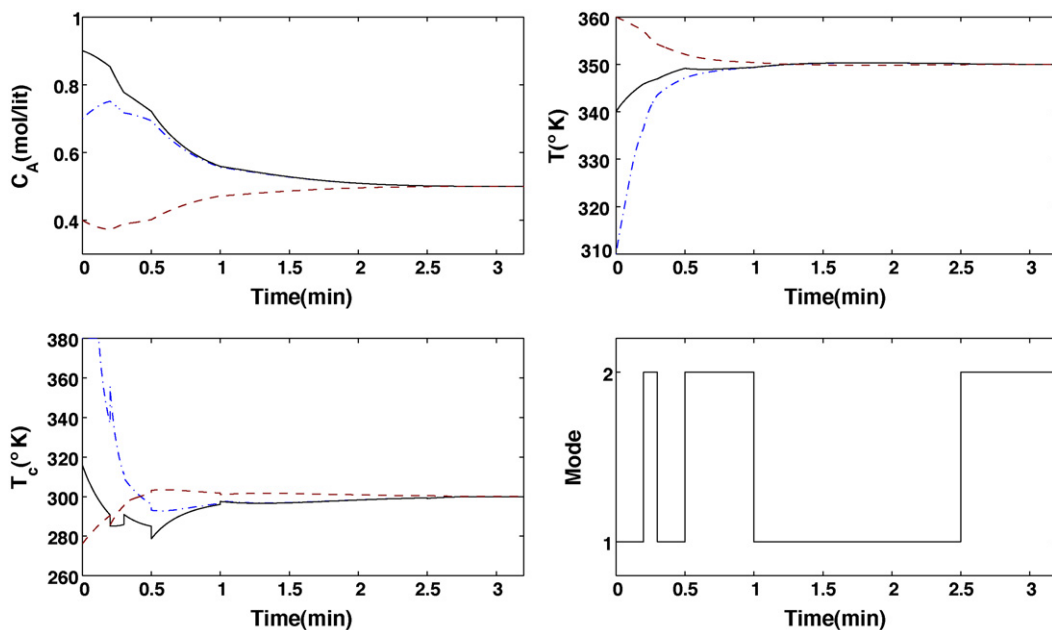


Fig. 3. States and control trajectories for $[C_{A0}, T_0] = [0.9, 340]$ (solid lines), $[C_{A0}, T_0] = [0.7, 310]$ (dash-dot lines), $[C_{A0}, T_0] = [0.4, 360]$ (dash lines) for an arbitrary switching signal.

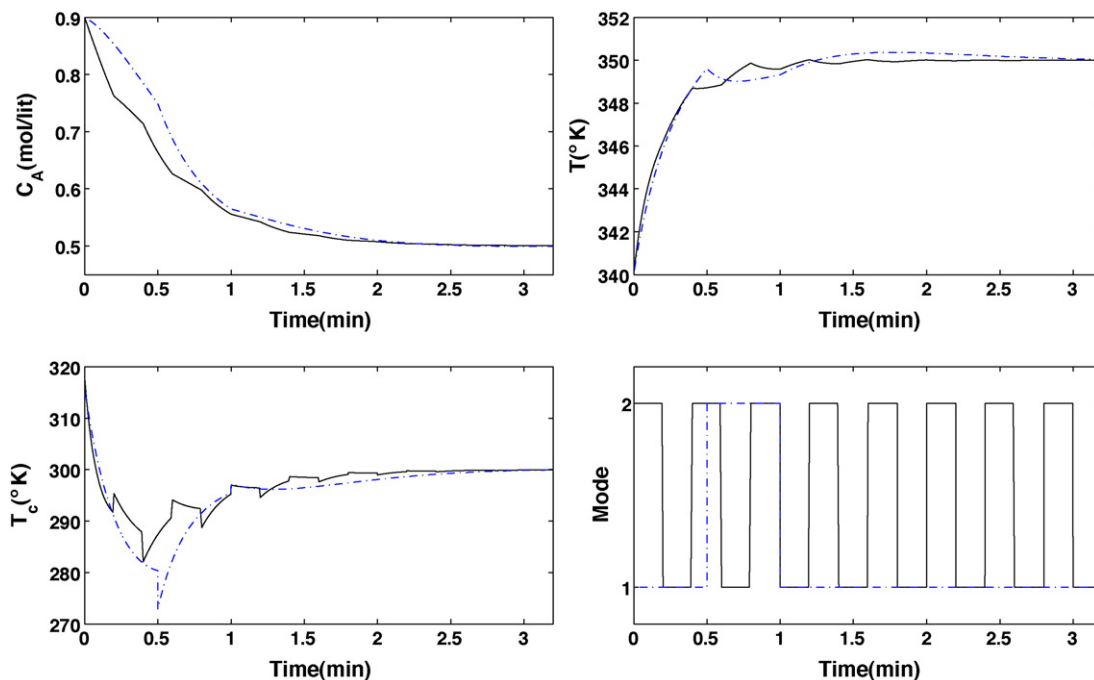


Fig. 4. States and control trajectories for initial states $[C_{A0}, T_0] = [0.9, 340]$ and under two different arbitrary switching signals.

our choice for the simulation. These gains guarantee that the SLS and consequently the SNS become globally stable under arbitrary switching signal.

4. Results and discussion

The closed-loop system, which is shown in Fig. 2, consists of the dynamics of the CSTR, the modal linearizing and the linear stabilizing control laws which have been designed in Sections 3.1 and 3.2. According to the theories which have been presented in Section 3, the designed controller under arbitrary switching signal and from arbitrary initial condition, steer the states of the process to the desired point, $[0.5, 350]$. Moreover, at steady state, the temperature of the coolant stream is the desired value, which is 300 K in this simulation. Here we present the simulation results for some arbitrary initial conditions and switching signals to illustrate the behavior of the controlled system.

Fig. 3 shows the convergence of the states and the control signal to the desired values using three different initial conditions. It can be seen that at switching instances the convergence behavior varies slightly. But in a general sense, the controlled system remains stable under the illustrated switching signal. Fig. 4 shows the state evolution of the system for two other switching signals. Different switching signals results in different transient behaviors but the control objectives are satisfied in both the cases.

Note that in practice, the physical constraints of the system that limit the behavior of the system should be take into account. For example, the admissible range for coolant stream temperature is limited in practice. This range is assumed to be $270 \text{ K} < T_c < 380 \text{ K}$ in our simulation. According to this constraint, the initial values of the reactor temperature and the concentration of species A in the reactor should be limited to prevent actuator saturation.

It may be useful to note that in Section 3.2, the state feedbacks are designed in such a way that guarantee the existence of a CQLF. For the state feedback gains which have been presented in Section 3.2, the function

$$19z_1^2 + 10z_1z_2 + 2z_2^2$$

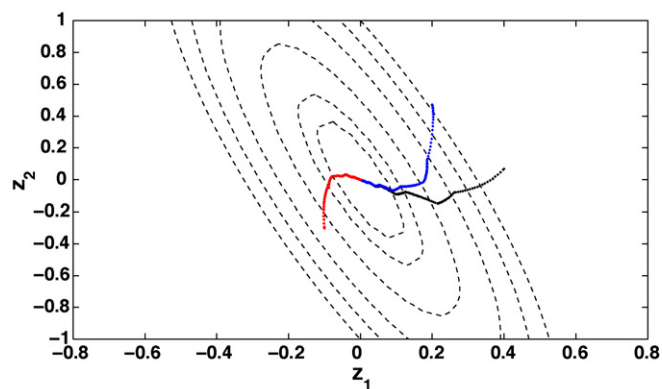


Fig. 5. Level curves of the common quadratic Lyapunov function (dash lines); state trajectories, from three different initial conditions and under a switching signal, which cross these curves.

is a CQLF. The level curves of this Lyapunov function are depicted in Fig. 5. Also, the state trajectories of the controlled system are illustrated using three different initial values and under an arbitrary switching signal. It can be seen that the trajectories cross the level sets such that the Lyapunov function value is always decreasing along the trajectories.

5. Conclusions

In this paper, a hybrid multi-loop controller design technique was developed for stabilizing a CSTR with a switched nonlinear dynamic. The inner loop was designed based on the new concept of modal state feedback linearization to construct an equivalent switched linear model for the system. The outer loop is a switched linear controller which guarantees global stability despite arbitrary switching of the CSTR modes, using the construction of a CQLF. Convergence of the process states to the desired values under arbitrary mode transitions was illustrated by simulation.

The main advantage of this technique is to simplify the procedure of designing a controller for an SNS. Unlike most of the existing approaches for stabilization of SNSs which leads to some challenging problems such as constructing Lyapunov functions for nonlinear dynamics [33–35] or estimating the region of attraction [32], this method reduces the problem of stabilization of an SNS to the stabilization of an SLS. The main limiting issue is that the state transformation should be identical for all subsystems. This fact restricts the application of our concept to second-order systems at this time. Extending the method to higher order and less restricted class of SNSs is the subject of future research. Also, additional work is required for the investigation of the effectiveness of the modal feedback linearization technique for other problems such as the stabilization of processes with more than two subsystems or with the tracking problem.

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